CHALLENGES IN THE SIMULATION OF THE DAYLIGHT DISTRIBUTION IN LATE ANTIQUE HAGIA SOPHIA

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Abstract

The deliberate utilisation of daylight has been discussed as an outstanding element of the design of Hagia Sophia in Istanbul starting with late antique sources, and continued through the centuries up to our day. Computational simulation allows to reconstruct this illumination. This - for the first time - offers a means to relate historical descriptions and modern hypotheses with daylight as a physical phenomenon, but requires a sound methodological basis to provide valid results. Bidirectional photon-mapping is demonstrated to predict the propagation of light in the architectural space, characterised by complexity with regard to its geometry as well as the optical properties of its boundaries. The latter must be reconstructed in their original condition, eliminating modifications of the original building composition as well as deterioration of the material and finishes of building elements. The description of the latter's irregular properties suggests data-driven modelling based on goniophotometric measurements. Either realistic or representative description of sky conditions based on climate data or high dynamic range imaging techniques is presented and critically discussed, since it provides descriptions of the contemporary conditions. Finally, the transfer of simulation results from the physical, photometric to the perceptual domain is necessary for any interpretation in the historical sciences, and to relate the quantitative outcomes to written accounts. The authors present methods and results from the exemplary application to the case of Hagia Sophia from their ongoing research. A research agenda is proposed to close the gap between archaeological evidence, advancements in the reconstruction and computational simulation of lighting, and its potential contribution to the historical sciences in arts and building.

Keywords: Daylight, reconstruction, visualisation, simulation, light scattering

GEÇ ANTİK DÖNEMDE, AYASOFYA'DA GÜN IŞIĞI DAĞITIMININ SİMÜLASYONUNDA TEKNİK ZORLUK

Özet

Gün ışığının planlı kullanımı İstanbul Ayasofya'nin önde gelen unsurlarından biridir. Ayasofya'nin tasarımındaki bu özellik, geç antik kaynaklardan başlayarak günümüze kadar, yüzyıllar boyunca tartışılagelmiştir. Görsel inceleme ve ölçüm yöntemleri, binanın aydınlatmasının niteliksel ve niceliksel olarak değerlendirilmesi açısından önemlidir. Ancak yüzyıllar boyunca devam etmekte olan restorasyon çalışmaları ve diğer modifikasyonlar nedeniyle, bu metotlar sadece binanın günümüzdeki halini incelememize olanak vermektedirler.

Rekonstrüksiyon ve simülasyon, tarihi ve kültürel araştırmalarda gittikçe daha sıklıkla ve yeni teknik bilgiler doğrultusunda gelişerek uygulanan deneysel bir metodolojik yaklaşımdır. Simülasyon ile aydınlatmanın rekonstrüksiyonunu sağlamak mümkündür. Simülasyon, tarihsel tanımları ve modern hipotezleri, fiziksel bir fenomen olarak gün ışığı ile ilişkilendirmek için bir araç sunar. Ancak, geçerli sonuçlar için sağlam ve güvenilirliği onaylanmış bir metodoloji gerekmektedir.

Ayasofya, karmaşık mekanın geometrisi ve yüzlerin optik özellikleri sebebi ile, simulasyonu oldukça zor bir binadır. Bu makalede kullanılan metodolojik yaklaşım üç ana madde altında açıklanabilir: i) geometrik özelliklerin doğru olarak modellenmesi; ii) yüzey malzemelerinin ışık yansıtma, ışık geçirgenliği ve ışık kırılması gibi optik özelliklerinin tüm karmaşıklığı ile karakterize edilebilmesi; ve iii) mekandaki ışık dağılımının fiziksel kurallara uygun olarak hesaplanabilmesi gerekmektedir.

Yeni geliştirilmiş yöntemler aracılığı ile Ayasofya'nın simulasyonunda amaç edinilen teknik özellikler sağlanmıştır. Darmstadt Teknik Universitesi'nde 2000 yılında başlayan araştırmalar kapsamında Ayasofya'nın 6. yüzyıldaki iç mekanının geometrisi ve iç yüzlerinin optik özellikleri simülasyon teknikleri ile modellenmiştir. Bu çalışmalar son 20 yılda ilerleyerek devam etmiştir ve ilk kez bu makalede detaylı olarak kullanılmıştır.

Geometrik modelleme, Ayasofya'nın tasarlandığı dönemdeki şekline dayalı özellikleri esas alınarak yapılmıştır. Tüm yapı daha büyük bir modelin parçası olabileceği gibi, yapı elemanlarının her biri, teker teker, gerektiğinde daha detaylı tanımlamalarla değiştirilebilecek bir modelleme esnekliği içinde kurgulanılmıştır.

Yüzey malzemelerinin ışık yansıtma, geçirgenlik ve kırılma gibi düzensiz optik özelliklerinin modellenmesi için gonio-fotometrik ölçümlerden elde edilen verilere dayalı bir modelleme geliştirilmiştir. Binaya giren gün ışığının ilk değişime uğradığı yüzey olan pencere camlarının özgün hali günümüzde var olmadığından, ölçümleri ve rekonstrüksiyonları doğrudan mümkün değildir. Ancak, antik dönemden beri tüm Roman imparatorluğunda üretilen göbekli, dökme düz ve silindir pencere cam örnekleri günümüze kadar ulaşmıştır. Bu çalışmada Ayasofya'nın pencerelerinde silindir pencere camı kullanıldığı ve camların Roma imparatorluğunda üretilen diğer camlar ile aynı teknikler kullanılarak üretildiği hipotezi üzerinden hareket edilmiştir. Bu kapsamda, silindir pencere camının optik özellikleri, Roma dönemi arkeolojik buluntuları esas alınarak, veriye dayalı ve çift yönlü ışık saçılma fonksiyonu kullanılarak analitik olarak modellenmiştir. Burada önemle belirtilmesi gereken unsur, Roma dönemine ait camların ışık geçirme ve saçılma özellik verileri, camların bugüne ulaştığı halleri üzerinden yapılan ölçümlere dayalıdır. Zaman aşımına bağlı olarak ışık geçirme özelliklerinin değişmediği varsayımı ile esas alınmıştır. Camların yüzey özelliklerinde yıllar içinde oluşabilecek bozulma etkisi göz önüne alınarak rekonstrüksiyonu, ileride incelenecek ayrı bir araştırma konusudur.

Ayasofya'nin geçirgen olmayan (opak) ve yarı saydam yüzeyleri, ışığı hem dağınık hem de yönlü olarak yansıtma özellikleri gösterdiğinden, oldukça karmaşıktır. Kubbelerde kullanılan cam mozaik, duvarlarda kullanılan farklı mermer çeşitleri, zeminde kullanılan süt beyazı renkli Marmara mermerleri ve kesik mermer kullanılarak duvarlara ve zemin yüzeylerinde gömülü yerleştirilen desenler (opus sectile) için geliştirilmiş modelleme teknikleri makalede detaylı olarak tartışılmıştır. Farklı mermer türlerinin bir kataloğu oluşturulmuştur. Altın mozaiğin karakteristik ışıltısı, mozaik yüzeylerin yayınık ışık saçılımı özelliklerinin ve her bir mozaik taşının geometrik modülasyonunun birleşik sunucu olarak ortaya çıkmakta olan bir yayınık yansımadır. Yayınık ışık saçılımları, cam yüzeylerin tek renkli yansımaları ile pürüzlü metal yüzeylerin renkli yansımalarının karışımından oluşan bir malzeme kombinasyonu ile tanımlanılmıştır. Her mozaik taşının geometrik modülasyonunu tek tek modellenmesi uygun olmadığından, parametrik modelleme tekniği kullanılmıştır.

Gün ışığı simülasyonda çoğunlukla tercih edilen kamera pozisyonundan ışık kaynağına geri yönlü ışın izleme yöntemi, Ayasofya'daki ışık dağılımının fiziksel kurallara uygun olarak hesaplanabilmesi için yeterli değildir. Bu çalışmada, iki yönlü, geri ışın izleme ve ileri foton-haritalama yöntemi ile, ışığın mimari alanda yayılmasının, mekan geometrisi ve iç yüzeylerin optik özellikleri göz önüne alınarak, tüm karmaşıklığı ile karakterize edilebileceği gösterilmiştir. Ayrıca, orijinal yapı kompozisyonundaki modifikasyonların yanı sıra, yapı elemanlarının ve bazı yüzey malzemelerindeki bozulmaların ortadan kaldırılması ile (örneğin, kırılmış ve çatlamış mermer yüzeylerin yıpranmamış hallerinin modellenmesi ile) yapının orijinal halinin incelenmesi mümkün olabilmektedir.

Gün ışığı simülasyonunda, geometrik ve yüzey özellikleri ile birlikte, gökyüzünün gün, saat, ay ve hava koşullarına bağlı olarak değişen aydınlık özellikleri de dikkate alınmalıdır. Gökyüzü koşulları, standart modeller kullanılarak (Uluslararası Aydınlatma Komisyonu (CIE) tarafından belirlenmiş açık, parçalı bulutlu ve bulutlu gökyüzü tanımlar ile), ya da meteorolojik verilere dayalı (matematiksel ve yüksek dinamik ölçekli fotoğraf-bazlı modeller ile) simüle edilebilmektedir. Gökyüzü modellerinin Ayasofya'nin orijinal halinin simulasyonu için kullanılmasında iki yaklaşım düşünülebilir. Birinci yaklaşımda günümüzde yapılan meteorolojik ölçümlerin 6. Yüzyıldaki gökyüzü koşullarını temsil ettiği varsayılabilir. İkinci yaklaşım için 6. yüzyıldaki iklim koşullarının rekonstrüksiyonu gerektir, ancak bu konuda yeterli veri bulunmamaktadır. Bu sebepten bu çalışmadaki simulasyonlarda günümüzdeki iklim koşulları esas alınmıştır ve gökyüzü koşulları iklim verilerine dayalı matematiksel modeller ve yüksek dinamik ölçekli görüntüleme teknikleri ile temsil edilerek değerlendirilmiştir.

Son olarak, simülasyon sonuçlarının fiziksel (fotometrik) boyuttan algısal alana aktarılması, tarihsel bilimlerdeki yorumlarla ve nicel değerlendirmelerle ilişkilendirmek için gerekli görülmüştür. Ayasofya'ya giren ziyaretçilerin fark ettiği gibi, gözün dış mekanlardaki ışık seviyelerinden iç mekandaki ışık seviyelerin uyum sağlaması zaman almaktadır. İnsanlardaki görsel sistem, bir görüntüyü keşfederken,

genel ışık seviyesine ve mekandaki parlaklık karşıtlığına uyum sağlar. Aydınlatma araştırmalarında yaygın olarak kullanılan ton haritalama operatörleri kullanılarak simülasyon sonucu oluşturulmuş fotometrik bilgilerin ışık uyumu göz önüne alınarak görselleştirilmesi yapılmış, böylece yapının fotometrik özellikleri yanında algısal olarak değerlendirilmesinin aydınlatma analizine sağladığı katkı tartışılmıştır.

Yazarlar, devam eden araştırmalarından Ayasofya vakasına uygulanmış örnek yöntemler ve sonuçlar sunmaktadır. Bu araştırmada sunulan metodolojik yaklaşımları, arkeolojik kanıtlar, aydınlatmanın yeniden yapılandırılması ve hesaplamalı simülasyonun gibi teknik bilimlerdeki gelişmeler ile sanat ve tarihsel bilimler arasındaki boşluğu kapatmak için bir araştırma gündemi olarak önermekteyiz.

Anahtar Kelimeler: Gün ışığı, rekonstrüksüyon, görselleştirme, simülasyon, ışık saçılması

INTRODUCTION

The role of light in the interpretation of pre-modern architecture is a recent, yet active field of research in the historical sciences:

> Light renders architecture's functionality. Throughout time, the conscious use of light has often underlined the various layers of meaning, from merely practical to utterly transcendental, that have been ascribed to built structures. (. . .) It was only recently that archaeologists and architectural historians have emphasised the role of light in the perception of premodern cultic and artistic spaces.[1]

Lighting, and the deliberate use of daylight in particular, has a tradition in the discussion of the late antique, imperial church of Hagia Sophia in Istanbul from its earliest descriptions to recent research.[2] While the subjective sentiment that the illuminated space conveyed in late antiquity cannot be concluded from the response of a modern observer,[3] the sensory stimulus affecting it can be quantitatively described by the

1 Vladimir Ivanovici and Daniela Mondini, eds., *Manipulating light in premodern times: Architectural, artistic, and philosophical aspects,* (Mendrisio, Switzerland: Mendrisio Academy Press, 2014), p. 12.

2 Silentiarios, Paulos and Procopius of Caesarea, "Paulos Silentiarios Beschreibung der Kirche der Heiligen Weisheit," *Bauten*, ed.: Otto Veh, (Berlin, Germany: de Gruyter), p. 306-358; Nadine Schibille, *Hagia Sophia and the Byzantine aesthetic experience* (Farnham, Surrey: Ashgate, 2014).

3 Vladimir Ivanovici, *Manipulating Theophany: Light and ritual in north adriatic*

response of the human eye to its environment. Measurements in terms of luminance (L, cd m^{-2})[4] and illuminance (E, lx)[5] aim at a quantitative description of this environment. However, photometric measurements can provide only a limited understanding of the role of daylight in Hagia Sophia:

1. Due to the alteration of the building after centuries of repairs and deliberate modifications, on-site measurements reflect the building only as of today.[6]

2. Instantaneous measurements are not representative but, due to the variable boundary conditions determined by changing sky conditions, a unique and possibly arbitrary snap-shot.

Evaluating the visual perception of a state in the past, or the intended design of Hagia Sophia requires a systematic approach to reconstruct the tangible form of the building. It has to simulate and visualise the intangible light propagation within its interior spaces, and to account for the response of the human eye.[7] Such *cultural heritage predictive rendering*[8], aims to predict the distribution of *illuminance on given*

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architecture (ca. 400–ca. 800), (de Gruyter, 2016), p. 6.

4 Mehlika Inanici, "Lighting analysis of Hagia Sophia," *Annual of Hagia Sophia Museum,* (Istanbul, Turkey: Hagia Sophia Museum, 2014), p. 128-202.

5 Nadine Schibille, "Light as an aesthetic constituent in the architecture of Hagia Sophia in Constantinople," *Manipulating light in premodern times: Architectural, artistic, and philosophical aspects,* ed.: Vladimir Ivanovici and Daniela Mondini, (Mendrisio, Switzerland: Mendrisio Academy Press, 2014), p. 31-43.

6 Andreas Noback et al., "Modelling the effects of daylight scattering by window glass: The case of sixth century Hagia Sophia in Istanbul," *20th congress of the Association Internationale pour l'Histoire du Verre*, Annales AIHV, under review (Istanbul, Turkey).

7 Andreas Noback, "Lichtsimulation in der digitalen Rekonstruktion historischer Architektur," *Der Modelle Tugend 2.0: Digitale 3D-Rekonstruktion als virtueller Raum der architekturhistorischen Forschung,* ed.: Piotr Kuroczyski, Mieke Pfarr-Harfst, and Sander Münster, (Heidelberg, Germany: Universitätsbibliothek Heidelberg, 2019).

8 Jassim Happa et al., "Cultural heritage predictive rendering," *Computer Graphics Forum* 31.6 (2012), p.1823-1836; Constantinos Papadopoulos, "Photorealism and Digital Reconstruction," *The Encyclopedia of Archaeological Sciences*, ed.: Sandra L López Varela (Malden MA, United States: Wiley Blackwell, 2018).

surfaces, or of *luminance in the field of view* of a given observer[9] by one of two approaches. Scene-Based Rendering (SBR) evaluates the interaction of a geometric description of the building with light. Since the model can represent a reconstruction, SBR lends itself to overcome the limitation of measurements to the present state of Hagia Sophia. Image-Based Rendering (IBR) considers imagery a function depending on independent variables such as the intensity and distribution of light sources. It allows to interpolate or extrapolate, and thereby addresses the second limitation of measurements.

A reconstruction guided by surveys, an analysis of the building[10], and considerations on building practice and mathematical knowledge at the time of its construction[11] – led to a surface model for daylight simulations employing SBR. Challenges emerging from the application of this detailed model in studies on the building's effects on its illumination, and the illumination's effects on the visual perception of the architectural interior, shall be discussed in context with extrapolation of photography as an exemplary application of IBR techniques to reveal the interplay of daylight and architecture in Hagia Sophia.

RECONSTRUCTION AND SIMULATION AS A CONTINUOUS PROCESS OF RESEARCH

The triad of reconstruction, visualisation, and simulation has been stated as a necessity for the research of lighting within historical sciences. Computer-aided *reconstruction* translates knowledge into complete and coherent models, and thereby reveals contradictions and research gaps.

9 Andrzej Piotrowski, "Representational function of daylight in the Katholikon of Hosios Loukas," *21st International Congress of Byzantine Studies* (London, United Kingdom, 2006); Jassim Happa and Alessandro Artusi, "Studying Illumination and Cultural Heritage," *Visual Computing for Cultural Heritage,* ed.: Fotis Liarokapis et al. (Cham, Switzerland: Springer, 2020), p. 23-42.

10 Robert L Van Nice, *Saint Sophia in Istanbul: An architectural survey* (Washington D.C., United States: Dumbarton Oaks Center for Byzantine Studies, 1965); Rowland J Mainstone, *Hagia Sophia: architecture, structure and liturgy of Justinians great church* (London, United Kingdom: Thames / Hudson, 1988).

11 Helge Svenshon and Rudolf H W Stichel, "'System of Monads' as design principle in the Hagia Sophia: Neo-Platonic mathematics in the architecture of late antiquity," *Nexus*, ed.: Sylvie Duvernoy and Orietta Pedemonte, VI Relationships between Architecture and Mathematics, (Turin, Italy: Kim William Books, 2006).

While *visualisation* provides a means to evaluate the model, *simulation* is an experiment in the virtual domain to test hypotheses. The outcomes of such experiments typically evoke further research questions and hypotheses. The iterative process of model alteration and experiment produces numerical and textual data as well as visual representations. Other than a linear process from reconstruction to visualisation[12], such continuous research not only requires accurate and valid representations and algorithms, but a platform for a dynamically evolving model over years. Laying the grounds for research on daylight in Hagia Sophia based on a reconstruction that did not exist yet, and on a first set of research questions that were known not to be the last, a hierarchical, dynamic model design was proposed at the turn of the millennium. Lending concepts and tools from the distributed development of open source software, the model was to integrate the geometric description of the reconstruction with documentation and simulation logic. Addressing the computational resources at the time, as well as to allow for continuous refinement, the model was organised as a tree-structure of building elements. As well as the entire model could become part of a larger model, each of its leafs could be replaced by refined structures. This open-ended design was implemented as a convention for a structure of files and directories. These are evaluated by a set of scripts, that assemble the model from its parts and run simulations and visualisations in a replicable manner[13]. Distributed access, recording and documentation of any modification, was provided by a revision control system.

The model relies on a set of shared routines, implemented on top of Make, a software that controls inter-dependent processes in software development. Definitions of geometry, as well as of other properties of the simulation model happen on the highest highest possible level of the building, and can get overwritten as refinements (Figure 1). This allows to start with a simple model, e.g. one geometric object (leaf). With each iterative refinement, objects are replaced by nodes linking to leafs (building parts) at the next lower level, forming a tree of theoretically unlimited complexity. Since the degree of refinement was initially unknown, branches of the tree developed different depths. Ultimately, e.g. each marble tile with its attributes became an object in this model.

12 Happa et al., "Cultural heritage predictive rendering"; Papadopoulos, "Photorealism and Digital Reconstruction."

13 Lars Oliver Grobe, "Theophanu - Scenebuilding environment," December 2, 2007.

Parts of the software tools employed in the initial design have been replaced. The choice of simple surface-meshes to store geometry was made due to a lack of standardised file formats at the time, motivating proposals such as Cultural Heritage Markup Language (CHML)[14]. Yet, the model's application in research on daylight in Hagia Sophia for about two decades, and its outcomes have proven its usefulness.[15]

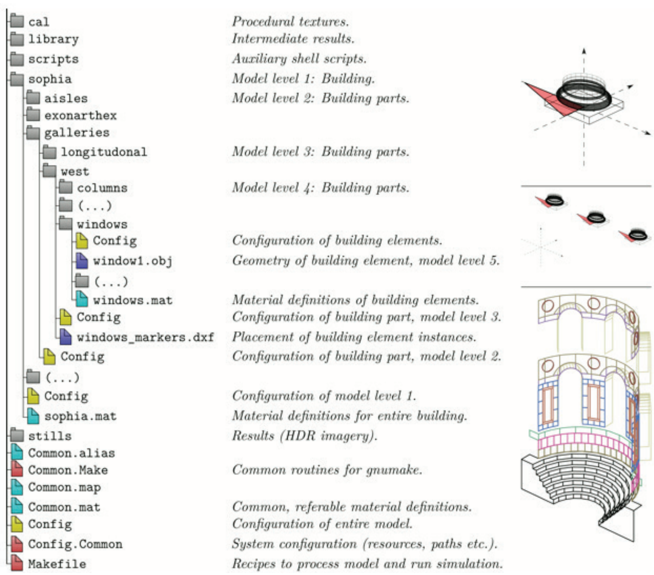

Figure 1. Left: Directory structure of the hierarchical simulation model with level-wise configuration (yellow), geometry (blue), descriptions of scattering properties (cyan) and processing instructions (red). Right: Instances of a building element (top) arranged on building part level (middle), and wall of the apse assembled from panel elements (bottom).

14 Oliver Hauck and Piotr Kuroczyski, "Cultural Heritage Markup Language – How to record and preserve 3D assets of digital reconstruction", *CHNT20 International Conference on Cultural Heritage and New Technologie* (Vienna, Austria, 2016).

15 Lars Oliver Grobe et al., "Das Licht in der Hagia Sophia – Eine Computersimulation," *Byzanz - das Römerreich im Mittelalter,* ed.: Falko Daim and Jörg Drauschke (Mainz, Germany: RGZM, 2010), p. 97-111; Oliver Hauck et al., "Computing the holy wisdom," *Scientific Computing and Cultural Heritage*, ed.: Hans Georg Bock, Willi Jäger, and Michael J Winckler (Berlin / Heidelberg, Germany: Springer, 2013), p. 205-216; Andreas Noback et al., "Hagia Sophias sixth century daylighting," *International Hagia Sophia Symposium – Architecture and Preservation,* In preparation (2020).

REPRESENTING VARIABLE SKY CONDITIONS

Hagia Sophia's geographic location (41.01° North, 28.98° East) determines the local sun-path, that comprises the apparent positions of the sun for any time of the year (Figure 2 left). Besides *direct sunlight* from these known directions, scattered *daylight* contributes to the incident distribution of daylight (Figure 2 right). Such scattering, and thereby the hemispherical distribution of daylight, is affected by the apparent sun position as well as weather conditions.

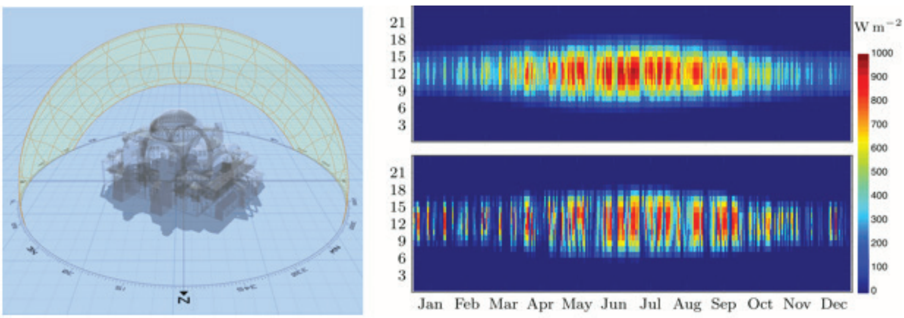

Figure 2. Left: Sun-path describing all possible apparent sun positions at the location of Hagia Sophia. Right: Frequency of hourly global horizontal irradiance (EGHI, top) and direct normal irradiance (EDNI, bottom) from meteorological observation in Istanbul

Synthetic sky models return the distribution of daylight as a function of time, or sun position, and a categorical description of the sky condition, e.g. as overcast, clear, or intermediate (Figure 3)[16]. They are employed to evaluate buildings under identical conditions to achieve comparability. While the position of the sun is defined, it is up to the researcher to select sky condition as appropriate for the question a simulation shall address. Idealised sky conditions allow to study extreme, but often not typical cases, that are often governed by intermediate conditions.

16 Y Uetani et al., *Spatial distribution of daylight – CIE standard general sky,* Standard ISO 15469:2004 (CIE S 011/E:2003), Reviewed 2018 (ISO / International Commission on Illumination CIE TC 3-15, 2004).

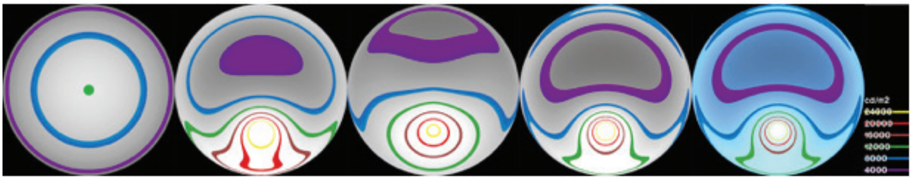

Figure 3. Sky luminance distributions (21st March, Istanbul) with overlaid contours bands. Left to right: Overcast , clear, and intermediate sky with sun; Perez model based on diffuse and direct irradiance, and enriched with colour information.

Describing atmospheric scattering by a set of physically meaningful parameters allows to reflect instantaneous sky conditions. Commonly applied models rely on few measured variables such as direct and diffuse irradiance, and infer the hemispherical luminance distribution of the sky. With available weather data[17], the sky distribution in terms of luminance[18] and even colour[19] can be modelled for any time that is covered by the records (Figure 3). Statistical analysis of the recorded data further allows to define representative Typical Meteorological Years (TMYs). As long as the mechanisms that determine sky conditions are invariant, such TMYs lend themselves to assess buildings in the past, when no weather records exist. However, such application of TMY would be rendered invalid if changes of the climatic conditions occurred. The reconstruction of daylight therefore requires further research in past changes of atmospheric conditions.

SCENE-BASED DAYLIGHT SIMULATION

1. Windows and Glass in Daylight Simulation

The reconstruction of Hagia Sophia before its later alterations and repairs reflects the presence of windows, that form the only apertures to admit daylight. Its distribution within the building can however not be described

17 ASHRAE, *International weather for energy calculations (IWEC Weather Files) users manual and CD-ROM*, EnergyPlus website, Atlanta, United States, 2014.

18 Richard Perez et al., “All-weather model for sky luminance distribution—preliminary configuration and validation,” *Solar Energy* 50.3 (1993), p. 235-245.

19 Jan Wienold and Aicha Diakite-Kortlever, *Making simulations more colorful: Extension of gendaylit to create a colored sky*, International Radiance Workshop, Loughborough, United Kingdom, 2018.

solely by the geometric reconstruction. The artisanal techniques of Roman glass production, that provided the window panes for the building, cause characteristic surface structures and inclusions that scatter light[20]. The chemical composition affected the spectrum of transmitted daylight. The optical properties of the original window glass must have caused three effects on the illumination of Hagia Sophia:

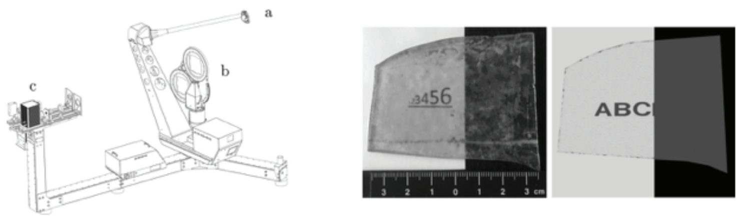

Figure 4. Left: Gonio-photometer comprising robotic arm with detector (a) aimed at sample (b), that is illuminated by light source (c). Image courtesy pab advanced technologies Ltd. Fragment of window glass from Ephesos (center) and its data-driven model (right).

1. Light transmitted through the windows was diffused. *Forward-scattering* maintained pronounced shadows and bright regions when transmitted sunlight reached diffusely reflecting surfaces. However, the diffusing effect forms soft shadows and contours of these *pools oflight*.[21]

2. Exposed to direct sunlight, the window panes appeared bright, due to a significant amount of *diffuse scattering* toward the observer even if the sun was not in the field of view. This effect can be imagined as a distinct glow of the window panes, which effectively became light sources.

3. Although the human eye adapts to the predominant light spectrum, colour perception in the interior was affected by the spectral modulation of daylight by the window panes.

Even if the particular properties of the original window glass in Hagia Sophia are not known, these effects are fundamental for any artisanally produced glass. They can therefore be expected, although to an unknown degree, to have affected the visual perception of the interior

20 Lars Oliver Grobe et al., "Data-driven modelling of daylight scattering by Roman window glass," *ACM Journal on Computing and Cultural Heritage* 13 (2020).

21 Inanici, "Lighting analysis of Hagia Sophia."

during daytime, and must be accounted for in daylight simulation. It is important to stress that daylight simulation cannot compensate for the lack of archaeological evidence, but can predict plausible scenarios. As a first attempt to establish a range of plausible optical properties of window glass, exemplary specimen from different parts of the late Roman empire were assessed and modelled.

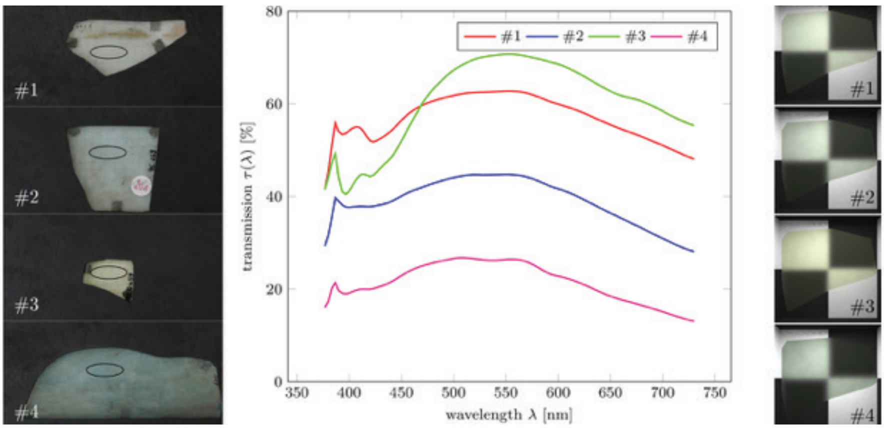

Figure 5. Four examples of Roman window glass (left) with corresponding transmission spectra (centre). Parametrisation of the Radiance trans model by the BSDF of glass #3, and the colours of #1 to #4 applied to it (right).

Light scattering is described by the Bidirectional Scattering Distribution Function (BSDF), an optical surface property that relates light transport between any pair of incident (toward the light source) and scattered (toward the receiver) directions. The BSDF of a fragment of Roman window glass from Ephesos, Turkey, was measured employing a scanning gonio-photometer[22] (Figure 4). The data-driven model implemented in Radiance allowed to accurately replicate the light scattering properties of the sample by the interpolation of the transmission distributions between the measured incident directions[23].

22 Peter Apian-Bennewitz, "New scanning gonio-photometer for extended BRTF measurements," *Proceedings SPIE*, 7792 Reflection, Scattering, and Diffraction from Surfaces II (San Diego, California, United States: SPIE, 2010).

23 Lars Oliver Grobe et al., "Daylight scattering by late antique window glass from Ephesus – Reconstructing the distribution of daylight in lost architecture," *CHNT24 International Conference on Cultural Heritage and New Technologies,* in press (Vienna, Austria).

Figure 6. Left: Acquisition of HDRI with colour chart for calibration Right: Textures modulating average colour of marble tiles and linings.

Figure 7. Heterogeneous decoration, e.g. opus sectile (left),were geometrically modelled so that different reflection properties could combined.

Rather than the direct compilation of measured BSDF into a data-driven model, least-square-fitting to measurements allows the parametrisation of the empirical trans modifier in Radiance. The approach was applied to one of a set of four samples from Northrine-Westfalia, Germany (glass #3 in the left column of Figure 5) based on the measurement for an incident off-normal angle of 20°. Furthermore, the spectral transmission of all four samples was measured (center) and combined with the scattering model of glass #3 (right column).

2. Complex Light Scattering Properties of Opaque Building Elements

Polished marble of a variety of colours as sheeting and pavement, and non-figurative gold mosaic covering vaults and domes, effect the dramatic appearance of Hagia Sophia that has been discussed in depth[24]. These opaque surfaces are perceived through the illumination they receive from the windows, as much as they relay and distribute light in the building.

24 Schibille, *Hagia Sophia and the Byzantine aesthetic experience.*

Despite their variety, the surface finishes share a distinct directional reflection. This specularity not only evokes a dramatic contrast by highlights due to mirror-like reflections, but also due to the apparent heterogeneity of surface finishes. Effects on the illumination of Hagia Sophia, as could be numerically described by the illuminance on its surfaces, are replicated by average properties. Vision, however, accounts for the polychromy within each marble sheet, the contrast of decorations such as the opus sectile, and the meso-structure formed by the mosaics. Replicating this level of detail in daylight simulation is particularly challenging and motivated different acquisition and modelling techniques.

Assuming that the observer distinguishes different marble colours and the texture within each tile, but not tiles of the same kind, a catalogue of marble types was established[25]. Photometrically and colourimetrically calibrated High Dynamic Range Imaging (HDRI) data for each identified type was acquired on site under diffuse illumination. The calibration relied on a chart featuring patches of known colour, that was imaged with each captured photograph (Figure 6, left). While the accuracy of this technique is limited, it allows to efficiently capture the visual detail at high resolution. The average colour and a monochrome, normalised texture (Figure 6, right) were derived for each marble type, and attributed to the corresponding sheets. The modulation by the texture adds a high level of visual detail, and ensures that the photometric and colourimetric averages are not affected. The approach was applied to surfaces where all reflection properties but reflectivity were assumed to be uniform.

Surfaces comprising different kinds of marble, e.g. the opus sectile (Figure 7, left), combine different reflection properties beyond colour and reflectivity, e.g. due to a different degree of polishing. For these building elements, the composition was replicated by geometric modelling (Figure 7, right). All reflection properties of the parts were defined individually[26].

25 Andreas Noback, "Tageslichtsimulation verlorener Gebäudezustände am Beispiel der Hagia Sophia Justinians," In preparation (PhD diss.), Technische Universität Darmstadt, 2020.

26 Hauck et al., "Computing the Holy Wisdom."

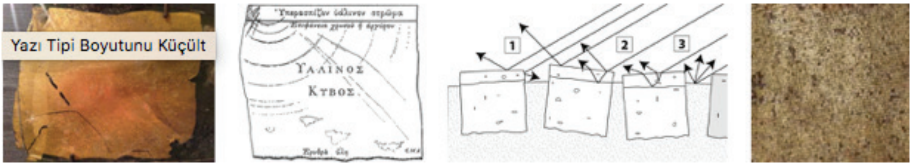

Figure 8. Left to right: The bulk glass-gold laminate is broken into tesserae. Light is deflected directionally by the front surface (1) or the embedded gold layer (2) of inclined tesserae, or diffusely by the mortar holding them (3). Together, the tesserea form a meso-structure.

The characteristic sparkle of gold mosaic is the combined effect of varying light scattering properties, and the geometric modulation of the reflecting surface due to the different inclination of the individual tesserae.

These are produced by breaking the bulk laminate of glass panes and embedded gold foil into small pieces, that are then set into mortar. Each tessera's highly specular reflection, either by the glass covering the front or by the covered gold foil, is directed to a slightly different direction. From a typical viewing distance thousands of tesserae are visible. Only a fraction of these are oriented so that they reflect light toward the observer, and stand out from the diffuse colour of the gold of the remaining mosaic, causing its distinct contrast (Figure 8)[27].

Since geometric modelling of the individual tesserae is not feasible for the entire building, a nested parametric model was developed[28]. It replicates light scattering by the meso-structure at different scales: The complex reflection by the individual tessera; the low-frequency, continuous perturbation of mortar holding the tesserae, the high frequency, discrete perturbation of each tessera's orientation, and the separation of tesserae by seams (Figure 9, left). Reflection by one tesserae was modelled by overlaying the sharp, monochrome reflection by a glass surface with the coloured reflection by a metal surface, the latter having a pronounced roughness. The result is a peak on a lobe in the BSDF, and confirmed by gonio-photometric measurement on a

27 Noback, "Tageslichtsimulation verlorener Gebäudezustände am Beispiel der Hagia Sophia Justinians"; Eugène Michel Antoniadi, *Ekphrasis tes Hagias Sophias* (Athens, Greece: Gregoriads, V / Huioi, 1983).

28 Noback, "Tageslichtsimulation verlorener Gebäudezustände am Beispiel der Hagia Sophia Justinians."

modern replication of a tessera (Figure 9, centre). The model can be applied to arbitrary surfaces and achieves a high degree of realism due to the added detail, without the increasing geometric complexity of the model (Figure 9, right). While it is possible to account for ornaments[29], vaults and domes were modelled only by the gold-glass tesserae reflecting the pending reconstruction of the Byzantine decoration for the model.

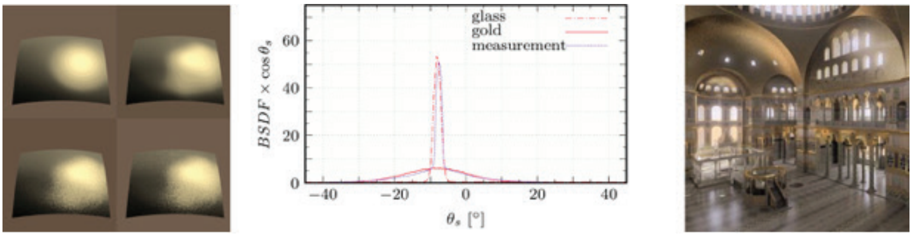

Figure 9. Left: Gold mosaic model combining reflection properties of the gold layer with large-scale deformations, inclination and seperation of tesserae. Centre: Combined reflection models of gold and glass approximate the measured BSDF of one tessera. Right: Meso-structured mosaic model applied to the reconstruction, seen from the West gallery.

3. Modelling Light Propagation in Hagia Sophia

Backward ray-tracing evaluates the BSDF at any point of intersection of an imagery light ray, starting at the view point, with the geometric description of a scene. It takes into account arbitrary light scattering properties, e.g. the specularity that is characteristic for the furnishing of Hagia Sophia. In the case of ideal reflection and transmission, e.g. by mirror-like or transparent surfaces, the algorithm is deterministic since each ray intersecting with geometry is propagated toward a maximum of two (reflected and transmitted) known directions. In the case of diffuse scattering, the contribution of each visible light source to the local illuminance is added. If indirect contributions by scattering between diffuse surfaces shall be accounted for, an infinite number of outgoing directions potentially lead to a source. This is accounted for by stochastic methods. *Distributed ray-tracing* generates random rays at each intersection event. The accuracy of the local illuminance estimate depends on the resolution, e.g. the number of sample rays at each intersection. This results in a non-linear increase

29 Andreas Noback, *Mosaics - Dealing with a precious material,* 11th International Radiance Workshop, Copenhagen, Denmark, 2012.

of computational effort (Table 1). Insufficient sampling causes typical artefacts such as *pixel noise*, illustrated by a view toward the entrance of Hagia Sophia in Figure 10. Radiance reduces the computational effort of the diffuse-indirect calculation by storing the local illuminance on the scene surfaces in its *ambient cache*. Subsequent ray intersections at nearby positions bypass the calculation and rather return the result of interpolation between known values. Only if a given variance is exceeded, further random samples are generated and continuously refine the cache during image generation. The high-frequency pixel noise is replaced by low-frequency interpolation artefacts (Figure 11), that diminishe with increasing sample resolution (Figure 12).

Pixel noise, and the artefacts of the ambient cache become obvious in the case of *caustics*. When light is scattered from specular toward diffuse surfaces, e.g. transmitted through the scattering window panes toward the entrance in the presented view, even a high number of samples is likely to miss out concentrated light sources such as the sun, or the apertures formed by the small window panes comprising Hagia Sophia's fenestration. This inefficiency of backward ray-tracing is the reason why the presented results all account for only one diffuse inter-reflection, and underestimate indirect illumination. Complex light paths such as caustics are better replicated when the direction of the simulation is reversed, e.g. from the light source toward the diffuse surface. The bidirectional *photon mapping* algorithm[30] stores the position where rays, emitted by the light source, collide with diffusely scattering geometry in *photon maps*, that provide a view-independent solution of local diffuse illuminance in the entire scene. During image generation, illuminance is estimated as the density of photons within a search radius, replacing the problematic stochastic sampling of distributed ray-tracing. Similar to the ambient cache, *photon mapping* introduces low-frequency *photon noise* that is evident in dark image regions, e.g. where the density of photons is low (Figure 13). Since the search radius has to increase until a given target of photons – the *bandwidth* – are found, *bias* deforms

30 Henrik Wann Jensen, *Realistic image synthesis using photon mapping* (Natick, MA, United States: AK Peters / CRC Press, 2001); Roland Schregle, "Daylight simulation with photon maps" (PhD diss.), Universität des Saarlandes, 2004; Lars Oliver Grobe, "The Radiance photon map for image-based visual comfort assessments with data-driven BSDF models of high resolution," *Journal of Building Performance Simulation* 1343 (2019).

sharp gradients toward a smooth transition. By indirect visualisation of global photons, noise and bias become less apparent (Figure 14). Both kinds of artefacts decrease with increasing numbers of photons in the scene (Figure 15). Although providing an efficient solution for complex light paths (Table 1), the broader application of photon mapping has been hindered by its complex parametrisation.

Figure 10. View rendered with ray-tracing at moderate sampling.

Figure 11. View as above, but employing ambient cache.

Figure 12. View as above, but with refined indirect-diffuse sampling.

Figure 13. View rendered with photon mapping and direct visualisation.

Figure 14. View as above, but indirect visualisation of global photons.

Figure 15. View as above, but number of photons eight times higher.

Table 1. Rendering parametrisation with resulting simulation times and artefacts (12 parallel processes, image resolution is 1600 x 900 pixels).

Illustration	Rendering technique	Parametrisation	Time	Artefacts
	Ray-tracing*			
Figure 10	without ambient cache	256 ambient divisions	1042 s	Pixel noise
Figure 11	with ambient cache	…	140 s	distinct splotchy artefacts
Figure 12	…	4096 ambient divisions	2051 s	moderate cloud-like artefacts
	Photon-mapping**			
Figure 13	direct visualisation	64M caustic, 16M global	33 s	pronounced photon noise, bias
Figure 14	indirect visualisation	8M caustic, 2M global	208 s	bias reducing contras
Figure 15	…	64M caustic, 16M global	219 s	low bias

* Reported results account for only one diffuse inter-reflection (-ab 1). The number of specular super-samples was constant (-ss 16). Ambient subdivisions (-as) were set to the number of ambient divisions (-ad) divided by 4. The ray weight (-lw) was limited to 4 divided by the ten times the number of ambient divisions.

** Reported results exclude time for photon map generation (1956 s for 8M caustic and 2M global photons, and 2272 s for 64M caustic and 16M global photons. Bandwidths for global and caustic photons were set to 40 and 60 respectively. 256 ambient divisions where set in the indirect photon visualisation.

Image-Based Daylight Simulation

IBR techniques cannot be directly applied to the reconstruction of lost built heritage, but they allow to inter- or extrapolate observations under varying boundary conditions. IBR techniques are not affected by the complexity of geometry and the light scattering properties of the building materials. *Image relighting* has been applied to reconstruct the appearance of isolated building elements, e.g. Byzantine gold mosaics, under their assumed original illumination[31]. For a given set of incident directions,

31 Eva Zányi et al., "Lighting and Byzantine glass tesserae," *Electronic Visualisation and the Arts Conference,* ed.: J P Bowen, S Keene, and L MacDonald (London, United Kingdom, 2007).

HDRI[32] was captured on site. The imagery can then be described as a function of the spatial distribution of incident luminance; and appearance under other luminous conditions can be replicated, if the latter are reconstructed e.g. by lighting simulation.

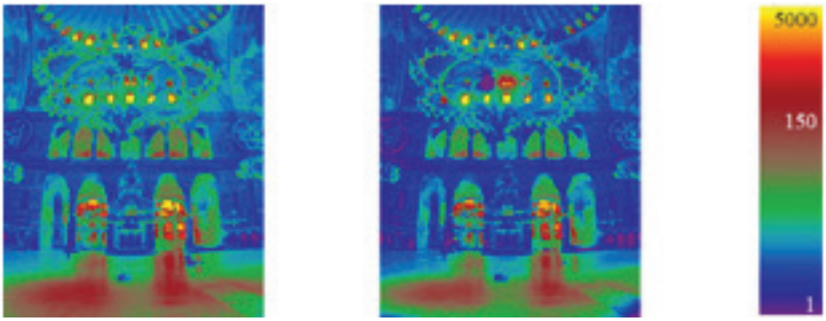

Figure 16. Left: Color-mapped luminance values acquired by HDRI photography. Right: Luminance dis- tribution under identical exterial conditions, calculated from Daylight Coefficients found by regression on synchronous HDRI acquisition.

A similar approach has been applied to the interior of Hagia Sophia. Given the difficulty of spatial and temporal measurements under controlled illumination, calibrated HDRI of the interior of the building, and the sky luminance distributions on a nearby roof terrace were acquired synchronously for a thorough analysis of daylight distribution of the interior under a set of naturally occurring sky conditions[33]. Furthermore, correlating the interior luminance distributions with the sky luminances allowed to reconstruct the Daylight Coefficients (DCs)[34], and to subsequently apply relighting techniques for the building interior (Figure 16). DCs are a unique property of the building site, geometry, and materials, and relate the average luminance of any sky region to its contribution to each pixel values in HDRI. DCs can be scaled through matrix operations to synthesise the appearance of the building interior

32 Mehlika Inanici, "Evaluation of high dynamic range image-based sky models in lighting simulation," *Leukos* 7 (2013), p. 69-84.

33 Inanici, "Lighting analysis of Hagia Sophia."

34 Mehlika Inanici, "Dynamic daylighting simulations from static high dynamic range imagery using extrapolation and daylight coefficient methodologies," *BS2013: 13th Conference of IBPSA,* ed.: Etienne Wurtz (Chambery, France, 2013), p. 3392-3399.

under any sky condition. The sky conditions can vary from idealised standard models for clear and overcast skies; data-driven models from HDRI captures and sky scans; or irradiance-based annual models that employ hourly data from meteorological stations.

Other IBR techniques that lend themselves to daylight simulation in Hagia Sophia due to the complexity of its geometry, and the irregular scattering properties of its surfaces, are machine learning and light field rendering. Given the dynamic nature of daylight, and the difficulty of collecting long-term data in the building, it is absolutely crucial to develop simulation techniques that can inform the spatial and temporal variability in the building.

Accounting for Human Response

As visitors entering Hagia Sophia notice, it takes a moment for the eye to adapt to the light levels inside the building. But human response is not limited to this adaption to a changing environment.[35] The visual system permanently adapts to contrast while exploring a scene. As the highest visual acuity of the eye is limited to its inner centre, a scene is permanently scanned by eye movement. During this process, adaption mechanisms are dynamically deployed, e.g. the constriction of pupils. Human vision responds to contrast in a nonlinear fashion, that can be expressed by the Weber-Fechner law[36] that formulates a logarithmic relationship between stimulus and perception. In effect, contrast in the mental image is reduced.

Common visual media, e.g. photographic prints or computer displays, are limited in contrast and absolute light levels compared to typical scenes. They do not cause contrast adaption, that can only be visualised for these media. A common approach in printing and display technologies is *gamma correction*, that implements the Weber-Fechner law. A more accurate visualisation can be achieved by tone-mapping. Tone-mapping operators commonly used in lighting research

35 Christian Bartenbach and Walter Witting, *Handbuch der Lichtgestaltung: Lichttechnische und wahrnehmungsphsychologische Grundlagen* (Wien, Austria: Springer, 2009), p. 78.

36 R.F. Schmidt, G. Thews, and F. Lang, *Physiologie des Menschen* (Berlin / Heidelberg, Germany: Springer, 2013), p. 210.

can reproduce the dynamic adaption during the perception of a scene.[37] Tone-mapping has to account for the maximal contrast of the medium under typical viewing conditions, which differ for prints and computer displays. HDRI-displays becoming increasingly affordable with the success of HDRI-television promise to mitigate the limitations in terms of dynamic range in the near future.

CONCLUSIONS

The authors believe that the presented results indicate the potential of SBR and IBR techniques in day- light simulation as a digital experiment in historical sciences. They hope to enter a fruitful dialogue with researchers from other disciplines, that have a long tradition in the discussion of daylight as an intangible but characteristic element of Hagia Sophia.

While the technical foundations for the application of computational techniques are laid, the simulation of daylight in the past remains a challenge. Following the propagation of light from the sun to the observer reveals a chain of uncertainties. The boundary conditions being defined by unknown sky conditions in the past, light was admitted through window panes that have not been preserved. Modelling these in their original condition is not possible, since measurement on archaeological finds include the effects of deterioration. Isolating these effects opens a promising field of research, and applies to any assumptions on optical properties attributed to computer-aided reconstructions. Experimental archaeology is considered one possible means to approximate former optical properties of building materials. Consequently, daylight simulation as a method to evaluate past lighting conditions must rely on plausible reconstructions not only of the geometric composition, but

37 Gregory Ward Larson et al., "A visibility matching tone reproduction operator for high dynamic range scenes," *IEEE Transactions on Visualization and Computer Graphics* 3.4 (1997): p. 291-306; Akiko Yoshida et al., "Perceptual evaluation of tone mapping operators with real-world scenes," *Human Vision and Electronic Imaging X*, ed.: Bernice E. Rogowitz, Thrasyvoulos N. Pappas, and Scott J. Daly, 5666 (SPIE, 2005), p. 192-203; Rafa Mantiuk et al., "High dynamic range imaging pipeline: Perception-motivated representation of visual content," *Human Vision and Electronic Imaging XII*, ed.: Bernice E. Rogowitz, Thrasyvoulos N. Pappas, and Scott J. Daly, 6492, Proceedings SPIE 649212 (San Jose, USA: SPIE, 2007).

also the light scattering properties attributed to this geometry as well as its boundary conditions.

Despite all attempts to improve accuracy and to account for alterations, daylight simulation in historical sciences has to handle, and to be aware of its uncertainties. In particular the partial knowledge on the optical properties of historic building materials, based on few detailed studies on individual specimens but covering only a fraction of the known range, asks for broader research. This calls for an interdisciplinary effort by researchers from archaeology, restoration, engineering, and sciences. Establishing plausible ranges of properties based on representative sample sets could foster the application of computational simulation, and would increase its credibility as a heuristic method in historical sciences.

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